SOME OLD AND SOME NEW THOUGHTS ON COMMUTANTS OF ANALYTIC MULTIPLICATION OPERATORS

CARL COWEN

Except in special circumstances, it is usually quite difficult to determine conditions that characterize which operators commute with a given operator. Such special circumstances include use of the spectral theorem for self-adjoint or normal operators and cases in which the operator in question has a rich point spectrum. The results in this latter situation come from the application of the fairly trivial observation that if A and B commute, the eigenspaces of A are invariant for B.

If $\mathcal{H}$ is a Hilbert space of analytic functions on the unit disk and T_z is the operator of multiplication by z, it is well known that the commutant of T_z is the collection of multiplication operators T_f where f is a bounded analytic function on the disk and $(T_f h)(z) = f(z)h(z)$.

In the 1970's and 80's, the question "Which operators on the Hardy space $H^2(\mathbb{D})$ commute with T_f for f a bounded analytic function on the disk?" was investigated. More recently, there has been interest in this question for the Bergman space $A^2(\mathbb{D})$. In this talk, an overview of the work of thirty years ago will be presented and we will consider this question for $f = B$, a finite Blaschke product, for T_B acting on the Bergman space. The question has wider consequences than might be expected and the answer is given in terms of a Riemann surface associated with the Blaschke product.